

University of Fribourg / Faculty of Science and Medicine / Department of
Mathematics

Groups of hyperbolic isometries and their commensurability

Edoardo Dotti

Let \mathbb{H}^n be the real hyperbolic n -space, $\text{Isom}(\mathbb{H}^n)$ its group of isometries and $\text{Isom}^+(\mathbb{H}^n)$ the index two subgroup of orientation preserving isometries.

In this thesis we establish some new algebraic commensurability conditions for the class of hyperbolic Coxeter groups. These are discrete groups in $\text{Isom}(\mathbb{H}^n)$ generated by finitely many reflections in the bounding hyperplanes of hyperbolic polyhedra all of whose dihedral angles are integral submultiples of π . At the basis is Vinberg's work associating to such a group a quadratic space (V, q) , which we call the Vinberg space.

We also exploit the Clifford matrix interpretation of $\text{Isom}^+(\mathbb{H}^n)$ and present our results, joint with S. Drewitz, about the realisability of right-angled hyperbolic polygons in any dimension.

For $n = 5$, we introduce a trace field for certain groups of complexified quaternionic 2×2 matrices. We show – in analogy to Kleinian groups in $\text{PSL}(2, \mathbb{C})$ – that this trace field is an algebraic number field.

Jury:

Prof. Dr. Anand Dessai, President of the jury

Prof. Dr. Ruth Kellerhals, Thesis supervisor

Prof. Dr. Vincent Emery, Referee

Prof. Dr. Jeffrey S. Meyer, Referee

Prof. Dr. Pavel Tumarkin, Referee